

19075478

INST0072 Logic and Knowledge Representation

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Assessment 1

Find cnf $\neg[(\neg\text{gamble} \wedge \text{work}) \rightarrow \text{happy}]$

- $\neg[(\neg\text{gamble} \wedge \text{work}) \rightarrow \text{happy}]$
- $\equiv \neg[\neg(\neg\text{gamble} \wedge \text{work}) \vee \text{happy}]$ **implication**
- $\equiv \neg\neg(\neg\text{gamble} \wedge \text{work}) \wedge \neg\text{happy}$ **de Morgan**
- $\equiv \neg\text{gamble} \wedge \text{work} \wedge \neg\text{happy}$ **Cancellation**

cnfset $\neg[(\neg\text{gamble} \wedge \text{work}) \rightarrow \text{happy}] = \{\neg\text{gamble}, \text{work}, \neg\text{happy}\}$

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Show that $\text{KB} \models (\neg\text{gamble} \wedge \text{work}) \rightarrow \text{happy}$

By the Resolution Soundness and Completeness theorem it is sufficient to show that:

$\text{KB} \cup \text{cnfset } \neg[(\neg\text{gamble} \wedge \text{work}) \rightarrow \text{happy}] \vdash_{\text{res}} \perp$

A derivation of this is as follows:

- (1) $(\neg\text{rich} \vee \neg\text{healthy} \vee \text{happy})$, ✓ by assumption from $\text{KB} \cup \{ \neg[(\neg\text{gamble} \wedge \text{work}) \rightarrow \text{happy}] \}$
- (2) $(\text{stressed} \vee \text{healthy})$, ✓ by assumption from $\text{KB} \cup \{ \neg[(\neg\text{gamble} \wedge \text{work}) \rightarrow \text{happy}] \}$
- (3) $(\text{gamble} \vee \neg\text{stressed})$, ✓ by assumption from $\text{KB} \cup \{ \neg[(\neg\text{gamble} \wedge \text{work}) \rightarrow \text{happy}] \}$
- (4) $(\neg\text{work} \vee \text{gamble} \vee \text{rich})$, ✓ by assumption from $\text{KB} \cup \{ \neg[(\neg\text{gamble} \wedge \text{work}) \rightarrow \text{happy}] \}$
- (5) $(\neg\text{gamble})$ ✓ by assumption from $\text{KB} \cup \{ \neg[(\neg\text{gamble} \wedge \text{work}) \rightarrow \text{happy}] \}$
- (6) (work) ✓ by assumption from $\text{KB} \cup \{ \neg[(\neg\text{gamble} \wedge \text{work}) \rightarrow \text{happy}] \}$
- (7) $(\neg\text{happy})$ ✓ by assumption from $\text{KB} \cup \{ \neg[(\neg\text{gamble} \wedge \text{work}) \rightarrow \text{happy}] \}$
- (8) $(\text{healthy} \vee \text{gamble})$ ✓ by (2), (3), resolution
- (9) (healthy) ✓ by (5), (8), resolution
- (10) $(\neg\text{rich} \vee \text{healthy})$ ✗ by (7), (1), resolution
- (11) $(\neg\text{stressed})$ ✓ by (5), (3), resolution
- (12) $(\text{gamble} \wedge \text{rich})$ ✓ by (6), (4), resolution
- (13) $(\neg\text{rich})$ ✓ by (12), (5), resolution
- (14) $(\neg\text{work} \wedge \text{gamble})$ ✓ by (13), (4), resolution
- (15) (gamble) by (14), (6), resolution
- (16) \perp by(15), (5), resolution

THIS SHOULD BE $\neg\text{healthy}$

THIS SHOULD BE rich

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THE RIGHT IDEA, BUT BE CAREFUL WITH "T" ON INDIVIDUAL PROPOSITIONS

Using the domain of discourse $D = \{A,B,C,D,E\}$ and signature $\{\{ \}, \{ \}, \{\text{admiral}/1, \text{boat}/1, \text{captain}/2, \text{boss}/2\}\}$

Give one interpretation $I +$ that satisfies and another interpretation $I -$ that does not satisfy the following sentence:

$\forall x. [\text{admiral}(x) \rightarrow \forall y. (\text{boat}(y) \rightarrow \exists z. [\text{captain}(z, y) \wedge \text{boss}(x, z)])]$

THIS NEEDS TO BE 'B' BECAUSE 'C' NEEDS TO BE CAPTAIN OF THE BOAT

<p>I^+ that satisfies</p> <p>I^+ (admiral) = (A) ✓</p> <p>I^+ (boat) = (B) ✓</p> <p>I^+ (captain) = $\{(C, D)\}$</p> <p>I^+ (boss) = $\{(A, C)\}$ ✓</p> <p style="color: red; text-align: center;">x</p>	<p>I^- that does not satisfy</p> <p>I^- (admiral) = (A) ✓</p> <p>I^- (boat) = (B) ✓</p> <p>I^- (captain) = $\{(C, B)\}$ ✓</p> <p>I^- (boss) = $\{(A, E)\}$ ✓</p>	<div style="border: 1px solid red; border-radius: 50%; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center; margin: 0 auto;"> 4 </div>
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Convert the sentence $\forall x. [\text{admiral}(x) \rightarrow \forall y. (\text{boat}(y) \rightarrow \exists z. [\text{captain}(z, y) \wedge \text{boss}(x, z)])]$ into prenex normal form:

$\forall x. [\neg \text{admiral}(x) \vee \forall y (\neg \text{boat}(y) \vee \exists z. [\text{captain}(z, y) \wedge \text{boss}(x, z)])]$ ✓ **Eliminating \rightarrow**

$\forall x \forall y \exists z. [\neg \text{admiral}(x) \vee \neg \text{boat}(y) \vee [\text{captain}(z, y) \wedge \text{boss}(x, z)]]$ ✓ **Moving Quantifiers Out**

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6
3
4 +
5

TOTAL: $\frac{18}{25} = \underline{\underline{72\%}}$

GOOD WORK ON THE WHOLE, WITH JUST ONE OR TWO ERRORS - SEE ME IF YOU NEED MORE EXPLANATION.